Kuwait University | Math 101 | July 19, 2008
Math. & Comp. Sci. Dept. | Second Exam | Time: 90 min.

Calculators and Mobile Phones are not allowed.

1. Use differentials to approximate: $\sec(46°)$. **(3 Points)**

2. Find an equation of the normal line at $x = 0$ to the graph of

$$y^3 + y\sin(x) - \cos(xy^2) = 0.$$

(4 Points)

3. A spherical ball made in steel is heated. If the surface area A of the ball (in cm^2) after time t (in hours) is given by

$$A = \sqrt{t^2 + t - 4},$$

find the rate at which the radius of the ball is changing after four hours.

(4 Points)

4. a) State the Mean Value Theorem. **(1 Point)**

   b) Use the Mean Value Theorem to show that:

$$(3+x)^{\frac{1}{3}} < 2 + \frac{1}{2}(x-5), \text{ for every } x > 5.$$

(3 Points)

5. Let $f(x) = \dfrac{-2x}{x+1}$, and given that $f'(x) = \dfrac{-2}{(x+1)^2}$ and $f''(x) = \dfrac{4}{(x+1)^3}$.

   a) Find the vertical and horizontal asymptotes for the graph of f, if any.

   b) Find the intervals on which the graph of f is increasing and the intervals on which the graph of f is decreasing. Find the local extrema of f, if any.

   c) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the points of inflection, if any.

   d) Sketch the graph of f.

   e) Find the maximum and minimum values of f on [1,2].

(10 Points)

Math-101 / Second Midterm — July 19, 2008

1. $\sec(46^\circ) \cong \sec(45^\circ) + \sec(45^\circ)\tan(45^\circ)\,\frac{\pi}{180} = \sqrt{2} + \frac{\sqrt{2}\,\pi}{180}$.

2. At $x=0$: $y^3 - 1 = 0 \Rightarrow y = 1$.

$$3y^2 y' + y\cos x + \sin x\, y' + \sin(xy^2)\,[2xy\,y' + y^2] = 0$$

The slope of the tangent line at $(0,1)$ is $m = -\frac{1}{3}$

$\Rightarrow$ the slope of the normal line at $(0,1)$ is $m_\perp = 3$.

Therefore, an equation of the normal line is $y = 3x + 1$.

3. $A = 4\pi r^2$.

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{9}{8} = 8\pi \frac{1}{\sqrt{\pi}} \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{9}{64\sqrt{\pi}} \text{ cm/hour.}$$

$A = \sqrt{t^2+t-4}$

$$\frac{dA}{dt} = \frac{2t+1}{2\sqrt{t^2+t-4}}$$

At $t=4$: $A = 4$, $r = \frac{1}{\sqrt{\pi}}$

$$\frac{dA}{dt} = \frac{9}{8}$$

4. b) $f(x) = (3+x)^{1/3}$ on $[5,x]$.

- f is continuous on $[5,x]$
- f is differentiable on $(5,x)$, $f'(x) = \frac{1}{3(3+x)^{2/3}}$.

By M.V.T., $\exists\, c$ in $(5,x)$ such that

$$f'(c) = \frac{f(x) - f(5)}{x-5}$$

$$\Rightarrow \frac{1}{3}\cdot\frac{1}{(3+c)^{2/3}} = \frac{(3+x)^{1/3} - 2}{x-5}, \quad c>5, \quad \frac{1}{(3+c)^{2/3}} < 1$$

$$\Rightarrow \frac{1}{3}(x-5) + 2 > (3+x)^{1/3} \Rightarrow (3+x)^{1/3} < 2 + \frac{1}{2}(x-5)$$

5. a) V.A: $\lim_{x\to -1^{\mp}} \frac{-2x}{x+1} = \mp\infty \Rightarrow x = -1$ is a V.A.

H.A. $\lim_{x\to\pm\infty} \frac{-2x}{x+1} = -2 \Rightarrow y = -2$ is a H.A.

b) $f'(x) = \frac{-2}{(x+1)^2}$.

No local extrema

$-\infty \quad -1 \quad \infty$

Intervals	$(-\infty,-1)$	$(-1,\infty)$
Sign of f'	$-$	$-$
Conclusion	$\searrow$	$\searrow$

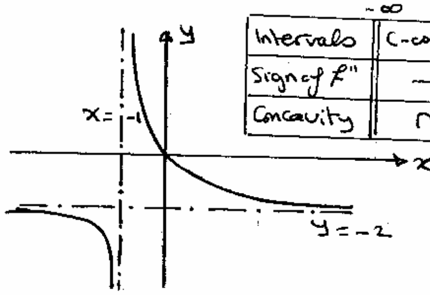
c) $f''(x) = \frac{4}{(x+1)^3}$

No P.I.

$-\infty \quad -1 \quad \infty$

Intervals	$(-\infty,-1)$	$(-1,\infty)$
Sign of f''	$-$	$+$
Concavity	$\cap$	$\cup$

d)



e) The max: $f(1) = -1$

The min: $f(2) = -\frac{4}{3}$